# The $B_s$ width difference beyond the Standard Model

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# Abstract

The Standard Model predicts a large width difference in the  $B_s$  system. New physics can contribute significantly to the mass difference. If this contribution is CP violating, this will always result in a reduction of the width difference. The analyses of measurements of the width difference using  $B_s$  decays into CP eigenstates have to be modified in the presence of new physics. We discuss how the width difference and the new CP violating phase in  $B_s$  mixing can be measured.

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#### I. INTRODUCTION

The physics of the  $B_s$  system is entering an exciting era, when experiments start exploring the range of mixing that is relevant to the Standard Model (SM) or to models of New Physics (NP). Recently, Dunietz [1] has studied the possibility of using untagged  $B_s$  samples for various measurements. The important ingredient in such an analysis is the large width difference that is expected in the  $B_s$  system. In some sense, the  $B_s$  system is similar to the neutral kaon system, where experiments study mass eigenstates ( $K_L$  and  $K_S$ ), rather than the  $B_d$  system, where experiments study the flavor eigenstates ( $B^0$  and  $\bar{B}^0$ ).

In [1], several ideas were put forward on how to measure the width difference,  $\Delta\Gamma$ , and how to extract the CP violating phase  $\gamma$ . However, the entire analysis was carried out in the framework of the SM. In this paper we investigate the implications of NP which can significantly affect the  $B_s$  mass difference,  $\Delta M$ . Phenomenological constraints on the relevant NP are rather weak. Moreover, various theories of flavor physics suggest that NP effects are more likely to appear in processes involving the heavy generation; the  $B_s$  system is unique in that it is the only neutral meson that is expected to exhibit mixing and does not involve first generation quarks.

We focus our attention on NP that contributes only to  $\Delta M$ . This is the case in most extensions of the SM: new, heavy particles cannot compete with the W-mediated tree level  $B_s$  decay amplitudes, and thus to a very good approximation all decay amplitudes are given by the SM. However, the new effects could even dominate over the highly suppressed box diagram mixing amplitude. Naively, one would think that since in such models the  $B_s$  decays are described by the SM, also the width difference is given by the SM. As we will discuss in this paper, this is incorrect.

The organization of the paper is as follows: in Section II we collect the relevant formalism. In Section III we discuss how new sources of CP violation that in general arise in NP models reduce  $\Delta\Gamma$ . In Section IV we describe how the width difference can be determined in a model independent way, and we analyze several methods to extract the new CP violating

phase using untagged  $B_s$  samples. We also argue that the unitarity angle  $\gamma$  can be extracted even in the presence of NP. Finally, Section V contains our conclusions.

#### II. FORMALISM

We start by collecting the relevant formulae and definitions. Most of them are well known [2]. An arbitrary neutral  $B_s$ -meson state,

$$a \mid B_s \rangle + b \mid \overline{B}_s \rangle$$
, (2.1)

is governed by the time-dependent Schrödinger equation

$$i\frac{d}{dt} \begin{pmatrix} a \\ b \end{pmatrix} = \mathbf{H} \begin{pmatrix} a \\ b \end{pmatrix} \equiv \left( \mathbf{M} - \frac{i}{2} \mathbf{\Gamma} \right) \begin{pmatrix} a \\ b \end{pmatrix}.$$
 (2.2)

Here M and  $\Gamma$  are 2 × 2 Hermitian matrices. CPT invariance guarantees  $H_{11} = H_{22}$ . The eigenstates of the mass matrix are

$$|B_L\rangle = p |B_s\rangle + q |\overline{B}_s\rangle, \qquad |B_H\rangle = p |B_s\rangle - q |\overline{B}_s\rangle,$$
 (2.3)

with eigenvalues

$$\mu_{L,H} = M_{L,H} - \frac{i}{2} \Gamma_{L,H} \,. \tag{2.4}$$

Here  $M_{L,H}$  and  $\Gamma_{L,H}$  denote the masses and decay widths of  $B_{L,H}$  and, by definition,  $M_H > M_L$ . In the SM  $\Gamma_L > \Gamma_H$  and then  $B_L$  is the short lived mass eigenstate. However, in the presence of NP this is not necessarily true. Therefore, we also label the mass eigenstates by  $B_{long}$  and  $B_{short}$ , where, by definition,  $\Gamma_{short} > \Gamma_{long}$ . We define

$$\Delta M \equiv M_H - M_L, \qquad \Delta \Gamma \equiv \Gamma_{short} - \Gamma_{long}, \qquad \Gamma \equiv \frac{\Gamma_L + \Gamma_H}{2}.$$
 (2.5)

The coefficient ratio q/p satisfies

$$\left(\frac{q}{p}\right)^2 = \frac{\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)}.$$
(2.6)

The width difference is

$$\Delta\Gamma = \frac{4|\operatorname{Re}(M_{12}\Gamma_{12}^*)|}{\Delta M}.$$
(2.7)

The experimental lower bound  $\Delta M/\Gamma > 8.8$  [3], implies  $\Delta M \gg \Delta \Gamma$  and consequently [2]  $|M_{12}| \gg |\Gamma_{12}|$ . Thus, to a very good approximation [2]

$$\Delta M = 2 |M_{12}|, \qquad |q/p| = 1.$$
 (2.8)

For a final state f we define the interference terms

$$\lambda \equiv \frac{q}{p} \frac{\langle f \mid \overline{B}_s \rangle}{\langle f \mid B_s \rangle}, \qquad \overline{\lambda} \equiv \frac{p}{q} \frac{\langle \overline{f} \mid B_s \rangle}{\langle \overline{f} \mid \overline{B}_s \rangle}. \tag{2.9}$$

We further assume  $|\langle f \mid B_s \rangle| = |\langle \overline{f} \mid \overline{B}_s \rangle|$  (namely, no CP violation in decay) and then  $|\lambda| = |\overline{\lambda}|$ . We always choose f such that  $|\lambda| \leq 1$ . The rapid time-dependent oscillations, which depend on  $\Delta Mt$ , cancel in untagged data samples [1]. We define

$$\Gamma[f(t)] \equiv \Gamma(B_{phys}(t) \to f) + \Gamma(\overline{B}_{phys}(t) \to f) , \qquad (2.10)$$

where  $B_{phys}$  ( $\overline{B}_{phys}$ ) denotes a time-evolved initially unmixed  $B_s$  ( $\overline{B}_s$ ). We have

$$\Gamma\left[f\left(t\right)\right] = \frac{\Gamma(B_s \to f)}{2} \left\{ \left(1 + |\lambda|^2\right) \left(e^{-\Gamma_L t} + e^{-\Gamma_H t}\right) + 2Re \,\lambda \left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right) \right\},$$

$$\Gamma\left[\overline{f}\left(t\right)\right] = \frac{\Gamma(B_s \to f)}{2} \left\{ \left(1 + |\lambda|^2\right) \left(e^{-\Gamma_L t} + e^{-\Gamma_H t}\right) + 2Re \,\overline{\lambda} \left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right) \right\}.$$
(2.11)

where we used  $|\lambda| = |\overline{\lambda}|$  and  $\Gamma(B_s \to f) = \Gamma(\overline{B}_s \to \overline{f})$ . Two limits are of particular interest. For decay modes which are specific of  $B_s$  or  $\overline{B}_s$ ,  $\lambda = \overline{\lambda} = 0$  (semileptonic decays are flavor specific;  $b \to c\overline{u}d$  decays are also flavor specific to a very good approximation),

$$\Gamma[f(t)] = \Gamma\left[\overline{f}(t)\right] = \frac{\Gamma(B_s \to f)}{2} \left\{ e^{-\Gamma_L t} + e^{-\Gamma_H t} \right\}. \tag{2.12}$$

For a final state that is a CP eigenstate,  $\lambda^* = \overline{\lambda}$  with  $|\lambda| = 1$ , and

$$\Gamma[f(t)] = \frac{\Gamma(B_s \to f)}{2} \left\{ e^{-\Gamma_L t} + e^{-\Gamma_H t} + Re\lambda \left( e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) \right\}. \tag{2.13}$$

#### III. $\Delta\Gamma$ IN THE SM AND BEYOND

 $\Delta\Gamma$  is produced by decay channels that are common to  $B_s$  and  $\overline{B}_s$ . In the SM,  $\Delta\Gamma$  is relatively large since the CKM unsuppressed parton decay  $b \to c\bar{c}s$  produces such final states. Two approaches have been used to actually estimate  $\Delta\Gamma$ . The first is a quark level calculation, where  $\Delta\Gamma$  corresponds to the imaginary part of the box diagram. Assuming factorization [4], and including QCD corrections [5], the result is

$$\frac{\Delta\Gamma}{\Gamma} \approx 0.20 \left(\frac{f_{B_s}}{210 \,\text{MeV}}\right)^2.$$
 (3.1)

In deriving (3.1),  $\Delta\Gamma = 2\Gamma_{12}$  has been used, which is justified when CP violation is neglected. This is a very good approximation in the SM. The second approach sums over exclusive channels [6]. Assuming factorization and Heavy Quark Symmetry relations between form factors,

$$\frac{\Delta\Gamma}{\Gamma} \approx 0.15. \tag{3.2}$$

The explicit dependence on the decay constants was not given in [6]. In general the width difference scales like the decay constant squared. Therefore, the use of the recently measured  $f_{D_s} = 273 \pm 39 \,\text{MeV}$  [7], instead of  $f_{D_s} = 230 \,\text{MeV}$  as used in [6] increases the estimate of  $\Delta\Gamma$ .

When NP contributes comparably to or dominates over the SM contribution to the  $B_s$  mixing, CP may be significantly violated. We now show that new CP violating contributions to the mixing always reduce  $\Delta\Gamma$  relative to the SM prediction. Eqs. (2.7) and (2.8) lead to

$$\Delta\Gamma = 2|\Gamma_{12}| |\cos 2\xi|, \qquad 2\xi \equiv \arg(-M_{12}\Gamma_{12}^*).$$
 (3.3)

Under the reasonable assumption that NP does not significantly affect the leading decay processes,  $\Gamma_{12}$  arises from  $\Gamma(b \to c\bar{c}s)$ . Consequently,  $2\xi$  is the phase difference between the total mixing amplitude and the  $b \to c\bar{c}s$  decay amplitude. In the SM,

$$\xi = \beta' \equiv \arg\left(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right) \approx 0, \tag{3.4}$$

and then  $\cos 2\xi = 1$  to a very high accuracy. With NP, new phases could be present, leading to  $\cos 2\xi < 1$ . This proves our statement.

The reduction of  $\Delta\Gamma$  can be understood intuitively as follows. In the absence of CP violation, the two mass eigenstates are also CP eigenstates. The large  $\Delta\Gamma$  is an indication that most of the  $b \to c\bar{c}s$  decays are into CP even final states. With CP violation, in the basis where the  $b \to c\bar{c}s$  amplitude is real, the mass eigenstates are no longer approximate CP eigenstates. Then, both mass eigenstates decay into CP even final states. Consequently,  $\Delta\Gamma$  is reduced.

The NP effects on the mixing amplitude  $M_{12}$  can be parameterized as

$$M_{12} = M_{12}^{SM} \left( 1 + ae^{i\theta} \right), \tag{3.5}$$

so that  $\xi$  is modified to (we approximate  $\beta' = 0$ )

$$2\xi = \arg\left(1 + ae^{i\theta}\right) = \tan^{-1}\left(\frac{a\sin\theta}{1 + a\cos\theta}\right). \tag{3.6}$$

The parameters a and  $\theta$  give the relative magnitude and relative phase of the NP contribution, i.e.  $a = |M_{12}^{NP}/M_{12}^{SM}|$  and  $\theta = \arg\left(M_{12}^{NP}/M_{12}^{SM}\right)$ . Phenomenological constraints do not exclude a > 1 [8], and  $\theta$  can have any value. Consequently, in the presence of NP,  $\xi$  could assume any value.

We do not study any particular NP model in detail, but mention here a few cases which predict (or can accommodate) large a and  $\theta$ . In models with vectorlike down type quarks, a tree level bsZ vertex is induced. The experimental bounds on  $BR(B \to X_s \mu^+ \mu^-)$  [9] and  $BR(B \to X_s \nu \bar{\nu})$  [10] allow for  $a \lesssim 0.25$  and arbitrary  $\theta$  [11]. In fourth generation models the t' box diagram can be large and a > 1 with arbitrary  $\theta$  is allowed [8,12]. In SUSY models without R parity, tree level sneutrino exchange (induced by the  $\Delta L = 1$  couplings) contributes to  $B_s$  mixing. Then, a > 1 with arbitrary  $\theta$  is allowed [13].

Finally, we mention the possibility of NP that affects the decay rates. An example would be models that give large  $BR(b \to sg) = O(10)\%$  [14]. Since  $b \to sg$  can produce CP eigenstates,  $\Gamma_{12}$  could change as well. In particular,  $\Delta\Gamma$  may become somewhat larger than

the SM prediction. For the rest of the paper, we restrict ourselves to the more reasonable case, namely when only  $M_{12}$  is significantly modified by the NP.

To conclude this section: If NP contributes significantly to the  $B_s$  mixing, the likely results are the enhancement of  $\Delta M$  and a suppression of  $\Delta \Gamma$ . If experiments are able to push the lower bound on  $\Delta M$  up, or the upper bound on  $\Delta \Gamma$  down, they might, in principle, be able to find NP.

### IV. MEASURING $\Delta\Gamma$ , $\xi$ AND $\gamma$

In this section, we examine how to measure  $\Delta\Gamma$  and the angles  $\xi$  and  $\gamma$  using untagged  $B_s$  data samples. In Ref. [1], several methods of how to measure  $\Delta\Gamma$  and  $\gamma$  have been analyzed in the framework of the SM. Here, we study how they have to be modified in the presence of NP and how they can probe it. We start by looking to the methods that were proposed to extract  $\Delta\Gamma$ . We show that by combining few measurements, both  $\Delta\Gamma$  and  $\xi$  can be extracted.

The first method is to fit the time dependent rate of flavor specific decays (see Eq. (2.12)) to two exponentials. This fit determines  $\Gamma_{long}$  and  $\Gamma_{short}$ , and therefore,  $\Gamma$  and  $\Delta\Gamma$ . This method is still valid in the presence of NP.

The second class of methods to measure  $\Delta\Gamma$  uses decays into CP eigenstates. (Unless otherwise specified, when we talk about CP eigenstates we refer to those that are produced by the  $b \to c\bar{c}s$  decay.) For concreteness, we focus on one method. We assume that  $\Gamma$  is known from either a fit to a flavor specific data samples, or from the average b hadrons lifetime [1]. Then, in the SM, the  $B_s$  lifetime measured using  $B_s$  decays into CP even final states determines  $\Gamma_{short}$ , and therefore,  $\Delta\Gamma$ . (Examples of such final states are  $D_s^+D_s^-$  and the CP even component of  $J/\psi\phi$  that can be obtained using transversity analysis [15].) As mentioned in [1], these methods are modified in the presence of NP. We now study this point in detail.

In a general NP model, when decays into CP eigenstates are dominated by the W

exchange tree diagram, we have

$$\lambda = \pm e^{2i\xi},\tag{4.1}$$

where, by convention, +(-) stand for a CP even (odd) final state. We emphasize that the phase  $\xi$  in (4.1) and (3.3) is the same phase. Then the decay rate into CP eigenstates (2.13) satisfies

$$\Gamma(B \to \text{CP even}, t) \propto \cos^2 \xi \ e^{-\Gamma_L t} + \sin^2 \xi \ e^{-\Gamma_H t}.$$
 (4.2)

For a decay into a CP odd state,  $\Gamma_L$  and  $\Gamma_H$  are interchanged.

A comment about discrete ambiguities is in order. In the SM the light state has a shorter lifetime and it is a CP even state [1]. However, in the presence of NP things may be different. When the NP contribution is large and negative ( $\cos 2\xi < 0$ ) the heavy state has a shorter lifetime, thus reversing the SM prediction. For  $\sin 2\xi \neq 0$  the mass eigenstates are no longer CP eigenstates, both eigenstates decay into CP even and CP odd states (see Eq. (4.2)). We emphasize, however, that always the longer lived state is "closer" to the CP odd state, namely, its decay width into CP odd states is larger than the decay width of the other mass eigenstate. Therefore, because  $\Delta M/M$  is tiny,  $\mathrm{sign}(\Gamma_L - \Gamma_H) = \mathrm{sign}(\cos 2\xi)$  is practically undetectable. Moreover, because  $2\xi$  always appears as an argument of a cosine function, also  $\mathrm{sign}(\sin 2\xi)$  cannot be determined using untagged  $B_s$  samples. (Note, however, that if the rapid  $\Delta Mt$  time dependent CP asymmetries in  $B_s$  decays will be measured, this sign will be determined, since asymmetries in modes governed by  $b \to c\bar{c}s$  can be used to extract  $\sin 2\xi$  [2].) In conclusion, in our analysis, there is a four fold ambiguity in the value of  $2\xi$ .

#### A. Combining flavor specific and CP eigenstates data

In principle, a three parameter fit of a decay into a CP even eigenstate can be used to measure  $\Gamma$ ,  $\Delta\Gamma$  and  $\xi$  using Eq. (4.2). Even if this cannot be done in practice, by comparing the measurements of  $\Delta\Gamma$  from flavor specific decays and CP eigenstate decays,  $\xi$  can be

measured. Experimentally, most of the data are expected to be taken for small  $\Gamma t$ . Then, using  $\Delta\Gamma t \ll 1$ , Eq. (4.2) becomes

$$\Gamma(B \to \text{CP even}, t) \propto e^{-\Gamma_+ t}, \qquad \Gamma_+ \equiv \left(\Gamma + \frac{\Delta\Gamma |\cos 2\xi|}{2}\right).$$
 (4.3)

Using  $\Gamma$  and  $\Delta\Gamma$  as measured from the flavor specific data, a one parameter fit to the decay rate gives  $\xi$ . Actually, such a fit determines

$$\Delta\Gamma_{CP} \equiv 2(\Gamma_{+} - \Gamma) = \Delta\Gamma |\cos 2\xi|. \tag{4.4}$$

By comparing it to the real width difference as measured using flavor specific data,  $\Delta\Gamma_{FS}$ , we get

$$|\cos 2\xi| = \frac{\Delta\Gamma_{CP}}{\Delta\Gamma_{FS}}.$$
 (4.5)

This method would be particularly useful if  $\xi$  is neither very small nor very large. For  $\xi \sim \pi/4$  the width difference becomes too small to be measured (see Eq. (3.3)). For  $\xi \sim 0$  the precision of the measurement should be very high. While we concentrate on one example, we emphasize that Eq. (4.5) is relevant to all the methods for measuring  $\Delta\Gamma$  using decays into CP eigenstates as suggested in [1].

#### B. Theory

Since  $\xi = 0$  in the SM, we get from Eq. (3.3)

$$\Delta\Gamma = \Delta\Gamma_{SM}|\cos 2\xi|. \tag{4.6}$$

We learn that, if we knew the SM prediction for  $\Delta\Gamma_{SM}$ , the measurement of  $\Delta\Gamma$  would allow for the determination of  $\xi$ . The problem is, of course, that we do not know how to calculate  $\Delta\Gamma_{SM}$  to a high accuracy. The box diagram calculation suffers from the uncertainty in  $f_{B_s}$  and, in any case, can be used only as an order of magnitude estimate since hadronic physics alters it. Similarly, there are large uncertainties in using the sum over exclusive states. However, we are able to get bounds on  $\xi$  even with these large uncertainties. To do that, we need only rough bounds on the SM range. The upper bound is rather safe, as  $\Delta\Gamma \leq \Gamma(b \to c\bar{c}s)$ . Based on the inclusive [16] and the exclusive [17] calculations, we can conservatively take an upper bound of  $BR(b \to c\bar{c}s) < 40\%$ . The lower bound is less reliable. Even if one can obtain a lower bound on  $BR(b \to c\bar{c}s)$  it would not help since we only know that  $\Delta\Gamma/\Gamma \leq BR(b \to c\bar{c}s)$ . However, from Eqs. (3.1) and (3.2) we can conservatively take  $\Delta\Gamma/\Gamma > 10\%$ . We believe then that it is safe to assume

$$10\% < \frac{\Delta\Gamma_{SM}}{\Gamma} < 40\%. \tag{4.7}$$

Therefore, a measurement of  $\Delta\Gamma$  will provided an upper bound on  $\xi$ :

$$\cos 2\xi > \left(\frac{\Delta\Gamma}{\Gamma}\right)_{\text{exp}} \frac{1}{40\%},\tag{4.8}$$

and similarly, any upper bound on  $\Delta\Gamma$  below the SM prediction, would give a lower bound on  $\xi$ :

$$\cos 2\xi < \left(\frac{\Delta\Gamma}{\Gamma}\right)_{\exp} \frac{1}{10\%} \,. \tag{4.9}$$

Note that  $\Delta\Gamma/\Gamma < 0.1$  would be evidence for NP; this is an unusual situation. Usually, NP enhances observables. In this case the SM is at the maximum and NP can only reduce  $\Delta\Gamma$ .

#### C. Time tag

It will be interesting if very late decaying  $B_s$  samples can be collected [1]. Then, all the short lived  $B_s$  have already decayed and the beam consists purely of the long lived state,  $B_{long}$ . Of course, for any useful measurement, large statistics is needed. The expected number of reconstructed  $B_s \to J/\psi \phi$  events is about  $1.2 \times 10^4$  at CDF in run II [18], and about  $3.8 \times 10^5$  per year at the LHC [19]. Assuming that this can be achieved,  $\xi$  could be measured using the time tag method.

From Eqs. (2.12) and (2.13) we get [1]

$$R(t) \equiv \frac{\Gamma[f(t)]}{\Gamma[g(t)]} \propto \left\{ 1 - Re \,\lambda \, \tanh\left(\frac{\Delta\Gamma \,t}{2}\right) \right\} \,, \tag{4.10}$$

where f is a CP even eigenstate and g is a flavor specific state. To enhance statistics, a sum over several CP even final states can be performed. Of course, if the time dependence can be traced,  $\Delta\Gamma$  and  $\xi$  can be extracted (recall,  $Re \lambda = \cos 2\xi$ ). In practice it might be easier just to make a cut to select events at large t, when  $\tanh (\Delta\Gamma t/2) \to 1$ . This can be thought of as an almost pure  $B_{long}$  beam. Then  $\xi$  can be obtained from the ratio

$$\frac{R(t \to \infty)}{R(t \to 0)} = \frac{\Gamma(B_{long} \to f)}{\Gamma(B_s \to f)} = (1 - |\cos 2\xi|) . \tag{4.11}$$

Finally, let us estimate the loss in statistics if this method is used. We define

$$P(t) \equiv \frac{N(B_{long}, t)}{N(B_{short}, t) + N(B_{long}, t)} = \left(1 + \exp(-\Delta\Gamma t)\right)^{-1}.$$
 (4.12)

where  $N(B_a, t) \propto \exp(-\Gamma_a t)$  is the number of  $B_a$  in the beam (a = long, short). In order to get a data sample with purity P for a given  $\Delta\Gamma$  we need to make the cut at  $t_{cut}$  such that

$$\Gamma t_{cut} > \left(\frac{\Gamma}{\Delta\Gamma}\right) \log\left(\frac{P}{1-P}\right).$$
 (4.13)

The loss in statistics is  $\exp(-\Gamma t_{cut})$ . For example, for P = 92% and  $\Delta\Gamma/\Gamma = 0.30$  one has to wait about 8 lifetimes, a loss of about  $3 \times 10^3$  in statistics. We learn that it may be possible to use the time tag method.

#### D. Measuring $\gamma$

In [1] several methods had been proposed for extracting

$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \tag{4.14}$$

Here we briefly describe the methods and show how they are modified in the presence of NP. More details on the methods, including the validity of the assumptions that are used, are given in Ref. [1].

There are basically two classes of methods. The first one is based on decays into CP eigenstates mediated by the  $b \to u\bar{u}d$  transition, e.g.  $\rho^0 K_S$ . When  $\Gamma$  and  $\Delta\Gamma$  are known, a

one parameter fit to the time dependence of untagged  $\rho^0 K_S$  events determines  $Re\lambda$  (see Eq. (2.13)). Assuming that the penguin contamination is small,

$$\lambda = \frac{q}{p} \frac{\langle \rho^0 K_S \mid \overline{B}_s \rangle}{\langle \rho^0 K_S \mid B_s \rangle} = -e^{2i(\xi + \gamma)}. \tag{4.15}$$

Then,  $Re\lambda = -\cos(2\xi + 2\gamma)$  and  $\xi + \gamma$  can be extracted.

The second class uses pairs of final states that are CP conjugate of each other and are mediated by the  $b \to c\bar{u}s$  and  $b \to u\bar{c}s$  transitions, e.g.  $D_s^-K^+$  and  $D_s^+K^-$ . When the total decay width,  $\Gamma(B_s \to f)$  is known [1], a three parameter fit to the time dependences of the decays into f and  $\bar{f}$  determines  $|\lambda|$ ,  $Re\lambda$  and  $Re\bar{\lambda}$  (see Eqs. (2.11)). In general,

$$\lambda = \frac{q}{p} \frac{\langle D_s^- K^+ \mid \overline{B}_s \rangle}{\langle D_s^- K^+ \mid B_s \rangle} = |\lambda| e^{i(\delta + 2\xi + \gamma)}, \qquad \overline{\lambda} = \frac{p}{q} \frac{\langle D_s^+ K^- \mid B_s \rangle}{\langle D_s^+ K^- \mid \overline{B}_s \rangle} = |\lambda| e^{i(\delta - 2\xi - \gamma)}, \tag{4.16}$$

where  $\delta$  is the strong phase. Then,  $Re\lambda = |\lambda|\cos(\delta + 2\xi + \gamma)$ ,  $Re\overline{\lambda} = |\lambda|\cos(\delta - 2\xi - \gamma)$  and  $2\xi + \gamma$  can be extracted.

We learn that when  $\Gamma$ ,  $\Delta\Gamma$  and  $\xi$  are known (from the combination of the flavor specific and the  $b \to c\bar{c}s$  CP eigenstates decays data) all these methods can be used to measure  $\gamma$ up to discrete ambiguities.

#### V. CONCLUSIONS

Various extensions of the Standard Model predict new significant contributions to the  $B_s$  mass difference. In general, such models introduce new sources of CP violation beyond the single phase of the CKM matrix. Still, in most cases, the  $B_s$  decay rates are described by the SM. Nevertheless, the width difference can be significantly reduced. Actually, such a reduction is an indication of CP violation: the large SM prediction for  $\Delta\Gamma$  is based on the fact that the decay width into CP even final states is larger than into CP odd final states. When new CP violating phases appear in the mixing amplitude then the mass eigenstates can sizably differ from the CP eigenstates, and both mass eigenstates are allowed to decay

into CP even final states\*. Consequently,  $\Delta\Gamma$  reduced.

Since in the SM the CP violating phase  $\xi$  is very small [2], ( $\xi \leq |V_{us}V_{ub}/V_{cs}V_{cb}| < 2.5 \times 10^{-2}$ ), a measurement of  $\xi \neq 0$  would be a clear evidence for CP violation beyond the SM. Actually,  $\xi \neq 0$  is the equivalent of  $\alpha + \beta + \gamma \neq \pi$ , which seems to be much harder to test experimentally [20]. Furthermore,  $\xi$  can be extracted using the  $B_s$  leading  $b \to c\bar{c}s$  decay modes (CKM unsuppressed). All these reasons explain why the measurement of  $\xi$  is very important. If the rapid  $\Delta Mt$  oscillation can be traced, the time dependent CP asymmetries in  $B_s$  decay modes mediated by  $b \to c\bar{c}s$  will measure  $\sin 2\xi$ . However, untagged  $B_s$  samples seem to be the best place for this measurement. We described several ways of determine  $\xi$  using untagged  $B_s$  samples:

• Compare the SM calculation (which has large uncertainties) to the measured  $\Delta\Gamma$ ,

$$|\cos 2\xi| = \frac{\Delta\Gamma}{\Delta\Gamma_{SM}}.\tag{5.1}$$

• Compare  $\Delta\Gamma$  as measured from decays into CP eigenstates, to  $\Delta\Gamma$  from flavor specific decays,

$$|\cos 2\xi| = \frac{\Delta\Gamma_{CP}}{\Delta\Gamma_{FS}}.$$
 (5.2)

• If a measurement at late times is possible, decay rates of a specific mass eigenstate can be measured, and then

$$|\cos 2\xi| = 1 - \frac{\Gamma(B_{long} \to \text{CP even})}{\Gamma(B_s \to \text{CP even})}.$$
 (5.3)

<sup>\*</sup>When |q/p| = 1 one can always choose a convention for CP transformation such that the mass eigenstates are CP eigenstates. But then, CP is violated by the decay amplitudes and both CP eigenstates decay into CP even states. Of course, the final result is the same, no matter what convention we use.

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